ENERGY DISSIPATION IN DILATING AND NONDILATING MEDIA

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Energy dissipation in an explosion in the ground was considered in [1, 2]. In this paper we analyze the distribution of the different forms of energy as a function of the time and distance, taking into account the possibility of decompaction of the medium (the phenomenon of dilation). It is based on the results obtained in [3] in which the problem of the motion of a shock front is solved and the behavior of dilating and nondilation media between the shock front and the expanding cavity is analyzed.

In the case of spherical symmetry and when there is no thermal conduction, the equation for the internal energy of a continuous medium has the form

$$\rho(\partial e/\partial t + u\partial e/\partial r) = \sigma_r \partial u/\partial r + 2\sigma_\varphi u/r, \tag{1}$$

where e is the amount of internal energy per unit mass; ρ, density of the medium; u, mass velocity; σ , tangential stress; t, time; and r, radius of the Euler coordinate. Equation (1) is considered at points of the medium situated between a cavity $a(t)$ and the shock front R(t).

The dilation equation, assuming spherical symmetry and that $\partial u/\partial r \leqslant 0$, has the form

$$\partial u/\partial r + 2u/r = \Lambda(u/r - \partial u/\partial r),$$

where Λ is the rate of dilation [4]. The condition of plasticity of the medium behind the wave front can be written in the form

$$\sigma_r - \sigma_\varphi = k + m(\sigma_r + 2\sigma_\varphi),$$

where k and m are known constants. Changing to Lagrangian coordinates (r_0, t) and using the last two relations, Eq. (1) can be written in the form

$$\frac{\partial e}{\partial t} = \left[(\alpha + n - 2)\, p - \frac{k\alpha}{3m}\right]\frac{u}{r\rho}. \tag{2}$$

where $n = (2 - \Lambda)/(1 + \Lambda)$; $\alpha = 6m/(2m + 1)$, $p = -\sigma_r$. The relation between the Lagrangian (r_0, t) and the Euler (r, t) coordinates is shown in Fig. 1, where curve 1 describes the motion of the shock front R(t), curve 2 describes the motion of the cavity $a(t)$, curve 3 corresponds to the change with time of the Euler coordinate r of the point of the medium with Lagrangian coordinate r_0 = const, and a_0 is the initial radius of the cavity. Integrating Eq. (2) along curve 3 (Fig. 1) we obtain

$$e_p(r_0, t) \equiv e(r_0, t) - e_f(r_0) = \int\limits_{t_1(r_0)}^{t} \left[(\alpha + n - 2)\, p - \frac{k\alpha}{3m}\right]\frac{u}{r\rho}\, d\tau, \tag{3}$$

where $t_1(r)$ is the function inverse to the function R(t), $e_p(r_0,t)$, energy of plastic deformation, $e_f(r_0) = e_s(r_0) + e_0$, energy on the front, $e_s(r_0)$, energy of shock compression, and e_0, internal energy of the medium in front of the front. Hence, the internal energy of a particle of the medium with Langrangian coordinate r_0 at the instant of time t is $e_p(r_0,t) + e_s(r_0) + e_0$. All the above energies relate to unit mass.

The energy of shock compression $e_s(r_0)$ is determined in the usual way from the laws of conservation of mass, momentum, and energy on the front. In our notation it has the form

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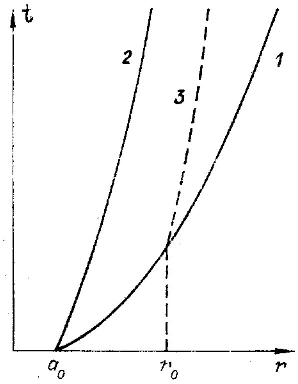

Fig. 1

$$e_s(r_0) = (\varepsilon_f/\rho_0)[p_0 + \varepsilon_f\rho_0\dot{R}^2(t_1(r_0))/2], \tag{4}$$

where ρ_0 and p_0 are the density and pressure, respectively, in the unperturbed medium, $\varepsilon_f = 1 - \rho_0/\rho_f$, jump in density on the front, ρ_f, density on the front, and $\dot{R}(t)$, velocity of the front.

To calculate (3) we must take into account the fact that

$$\int\limits_{t_1(r_0)}^{t} \frac{u(r_0,\tau)}{r(r_0,\tau)\rho(r_0,\tau)}\,d\tau = \begin{cases}(1/\rho - 1/\rho_f)/(2-n), & \text{if} \quad n \neq 2,\\ (1/\rho_f)\ln(r/r_0), & \text{if} \quad n = 2.\end{cases}$$

since $u(r_0, t) = \partial r(r_0, t)/\partial t$, while $\rho(r_0, t) = \rho_f(r_0/r)^{2-n}$ [3]. It follows from [3] that in the functions $p(r_0, t)$, $r(r_0, t)$ the time dependence is present in terms of the radius of the front $R(t)$: $p(r_0, t) \equiv p_1(r_0, R(t))$,

$$r(r_0, t) \equiv r_1(r_0, R(t)) = [(1-\varepsilon_f)r_0^{n+1} + \varepsilon_f R^{n+1}(t)]^{\frac{1}{n+1}}. \tag{5}$$

After replacing the variable in the integral from Eq. (3) for the energy of plastic deformation we obtain the following expression, which is convenient for numerical calculations:

$$e_p(r_0, t) = \frac{(\alpha+n-2)\varepsilon_f}{\rho_f r_0^{2-n}} \int\limits_{r_0}^{R(t)} \frac{p_1(r_0, q)}{r_1^{2n-1}(r_0, q)}\, q^n dq - \frac{k\alpha}{3m(2-n)}\left(\frac{1}{\rho} - \frac{1}{\rho_f}\right) \tag{6}$$

if the medium dilates ($n \neq 2$), and

$$e_p(r_0, t) = \frac{\alpha\varepsilon_f}{\rho_f} \int\limits_{r_0}^{R(t)} \frac{p_1(r_0, q)}{r_1^3(r_0, q)}\, q^2 dq - \frac{k\alpha}{3m\rho_f}\ln\frac{r}{r_0} \tag{7}$$

if the medium does not dilate ($n = 2$).

The kinetic energy, referred to unit mass, is $e_k(r_0, t) = u^2(r_0, t)/2$, where, as is well known [3],

$$u(r_0, t) = \varepsilon_f R^n(t)\dot{R}(t)/r^n(r_0, t). \tag{8}$$

Hence, all the energy per unit mass for a particle with Lagrangian coordinate r_0 at the instant of time t will be $e_p(r_0, t) + e_s(r_0) + e_k(r_0, t) + e_0$. The sum of the first two terms is the increment in internal energy, while the sum of the first three is the increment of the total energy with respect to the initial energy e_0 of the unperturbed state. The total energy $E_p(t)$, for plastic deformation, the total energy $E_s(t)$, for shock compression, and also the total kinetic energy $E_k(t)$ and the initial energy $E_0(t)$ up to the instant of time t are given, respectively, by

$$E_{p(s,k,0)}(t) = 4\pi\rho_0 \int\limits_{a_0}^{R(t)} r_0^2 e_{p(s,k,0)}(r_0, t)\, dr_0.$$

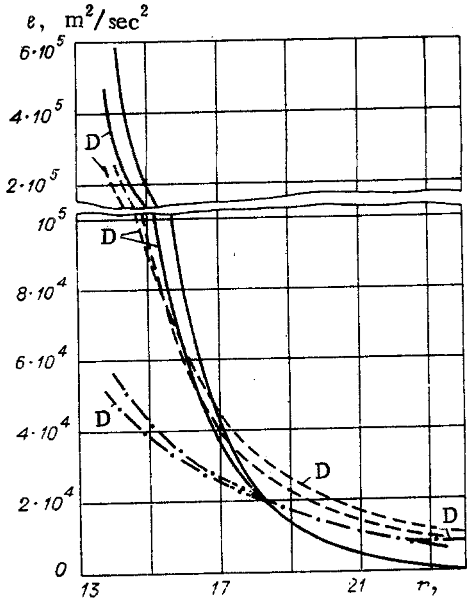

Fig. 2

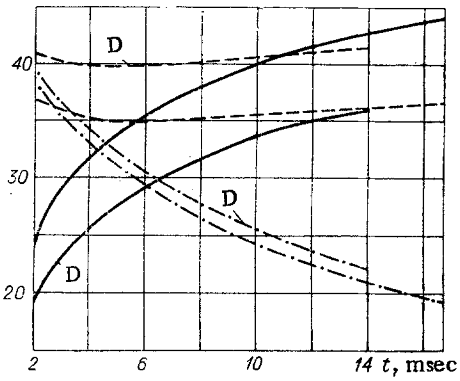

Fig. 3

The final equations have the form

$$E_p(t) = \frac{4\pi(\alpha+n-2)\rho_0\varepsilon_f}{\rho_f}\int_{a_0}^{R(t)} r_0^n \left\{\int_{r_0}^{R(t)} \frac{p_1(r_0,q)}{r_1^{2n-1}(r_0,q)}\, q^n dq\right\} dr_0 - \frac{4\pi k\alpha}{9m(2-n)}\left[(1-\varepsilon_f)a_0^3+\varepsilon_f R^3(t)-r^3(a_0,t)\right] \quad (9)$$

if n ≠ 2, and

$$E_p(t) = \frac{4\pi\alpha\rho_0\varepsilon_f}{\rho_f}\int_{a_0}^{R(t)} r_0^2 \left\{\int_{r_0}^{R(t)} \frac{p_1(r_0,q)}{r_1^{3}(r_0,q)}\, q^2 dq\right\} dr_0 - \frac{4\pi k\alpha}{9m}\left[\varepsilon\; R^3(t)\ln\frac{R(t)}{r(a_0,t)} - \frac{\rho_0}{\rho_f}a_0^3\ln\frac{r(a_0,t)}{a_0}\right] \quad (10)$$

if n = 2;

$$E_s(t) = (4/3)\pi\varepsilon_f p_0\left(R^3(t)-a_0^3\right) + 2\pi\rho_0\varepsilon_f^2\int_0^t R^2(s)\dot{R}^3(s)\,ds; \quad (11)$$

$$E_k(t) = 2\pi\rho_0\varepsilon_f^2 R^{2n}(t)\dot{R}^2(t)\int_{a_0}^{R(t)} \frac{r_0^2}{r^{2n}(r_0,t)}\,dr_0; \quad (12)$$

the last integral when there is no dilation is also taken for n = 2 and has the form

$$E_k(t) = 2\pi\rho_f\varepsilon_f^2 R^4(t)\dot{R}^2(t)\left(1/r(a_0,t)-1/R(t)\right). \quad (13)$$

$$E_0(t) = (4/3)\pi\rho_0\left(R^3(t)-a_0^3\right)e_0.$$

The quantities R(t), $p_1(r_0, R(t))$, required for calculations using Eqs. (4), (6), (7), and (9)-(13), are determined using the equation for R(t) from [3].

The total increment of all the internal energy $E_a(t) = E_p(t) + E_s(t)$ while $E(t) = E_a(t) + E_k(t)$ is the total increment of all the energy and should be equal to the work A(t) of the product of the explosion performed on the medium. The equation $A(t) = E_p(t) + E_s(t) + E_k(t)$ can be used as an objective check of the correctness of the calculations. We have the following expression for the work A(t) taking Eqs. (5) and (8) into account:

TABLE 1

Form of total energy, J	$\Lambda=0{,}14$	$\Lambda=0$	Form of total energy, J	$\Lambda=0{,}14$	$\Lambda=0$
E_p	$4{,}03\cdot10^{12}$	$4{,}88\cdot10^{12}$	$E=E_a+E_k$	$1{,}11\cdot10^{13}$	$1{,}14\cdot10^{13}$
E_s	$4{,}63\cdot10^{12}$	$4{,}10\cdot10^{12}$	A	$1{,}11\cdot10^{13}$	$1{,}14\cdot10^{13}$
E_k	$2{,}46\cdot10^{12}$	$2{,}40\cdot10^{12}$	$\|A-E\|$	$4{,}89\cdot10^{9}$	$6{,}12\cdot10^{9}$
$E_a=E_p+E_s$	$8{,}66\cdot10^{12}$	$8{,}98\cdot10^{12}$	$\frac{100\,\|A-E\|}{A}$	0,04%	0,05%

$$A(t)\equiv 4\pi\int_{a_0}^{r(a_0,t)} p(a_0,\tau)\,r^2(a_0,\tau)\,dr(a_0,\tau) = 4\pi\varepsilon_f\int_{a_0}^{R(t)} p_1(a_0,q)\left[(1-\varepsilon_f)\,a_0^{n+1}+\varepsilon_f q^{n+1}\right]^{\Lambda} q^n dq. \quad (14)$$

The problem of the propagation of a spherically symmetrical shock wave in a plastic medium can be solved in closed form if, in addition to the laws of conservation, we are given the change in the pressure in the cavity $p(a_0\ t)\equiv p_1(a_0,\ R(t))$. It is assumed that the exploding cavity expands adiabatically with an adiabatic constant γ [2, 3], i.e.,

$$p_1(a_0, R(t))=p_{k0}\left[a_0/r_1(a_0, R(t))\right]^{3\gamma},$$

where p_{k_0} is the initial pressure in the cavity. Then the integral in Eq. (14) is also taken for the work of the explosion products A(t) and we have

$$A(t)=\begin{cases}(4/3)\,\pi a_0^3 p_{k0}\left[1-(r(a_0,t)/a_0)^{3-3\gamma}\right], & \text{if } \gamma\neq 1,\\ 4\pi a_0^3 p_{k0}\ln\left(r(a_0,t)/a_0\right), & \text{if } \gamma=1.\end{cases}$$

In Figs. 2 and 3 and in Table 1 we give some results of calculations carried out for the following initial data: p_{k_0} = 62 kbar, p_0 = 0.25 kbar, ρ_0 = 2.5 g/cm³, a_0 = 7 m, ε_f = 0.2, k = 0.35 kg/cm², m = 0.45, γ = 1.5, Λ = 0.14 or Λ = 0. The letter D in Figs. 2 and 3 denote curves giving the relations taking dilation into account (Λ = 0.14). The continuous curves represent the energy of plastic deformation, the broken curves represent the energy of shock compression, and the dash–dot curves represent the kinetic energy. The results given in Fig. 2 and in the table relate to the instant of time t = 14 msec. At this instant of time when dilation occurs (Λ = 0.14) R = 24 m, a = 13.8 m, $\dot{R}$ = 636 m/sec, and the acceleration $\ddot{R}$ = 30.8 km/sec²; the corresponding numbers ignoring decompaction (Λ = 0) will be R = 23.4 m, a = 14.1 m, $\dot{R}$ = 605 m/sec, and $\ddot{R}$ = 29.5 km/sec².

Figure 2 shows the internal energies e_p, e_s, and e_k as a function of the Euler radius r. As can be seen, the increment in internal energy per unit mass $e_p + e_s$ is particularly large close to the exploding cavity. When the phenomenon of dilation is taken into account the slope of this curve is smoothed out somewhat.

Table 1 shows values of the total energy; $|A - E|$, absolute error of the calculation; and, $100\,|A-E|/A$, relative error. Figure 3 shows as a percentage the fraction of the different forms of the total energy as a function of time, i.e., graphs of the functions $100\,E_p(t)/E(t)$, $100\,E_s(t)/E(t)$, and $100\,E_k(t)/E(t)$. The calculations show that in this time interval the fraction of the energy of shock compression is approximately constant: 40-41% for a dilating medium and 35-37% for a nondilating medium. As the time increases the total fraction of the kinetic energy falls, and the total fraction of the energy of plastic deformation increases. Consideration of the dilation reduces the fraction of the energy of plastic deformation.

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